To: George Benson, President, College of Charleston  

Dear President Benson,

I request a sabbatical leave for the full 2013-2014 academic year, to pursue work described in the attached proposal.

At the beginning of the leave, I plan to visit the Mathematics Department at Penn State University, to interact and collaborate with the Geometry and Integrable Systems Group and with Professor Diane Henderson, faculty leader of the W.G. Pritchard Fluid Mechanics Laboratory. A letter of invitation by Professor Luen-Chau Li is included.

I propose to work on three research projects, two of which are supported by grant DMS-1109017, Collaborative RUI. Nonlinear Schrödinger Models in Fluid Dynamics: Rogue Waves and Vortex Filaments, awarded by the National Science Foundation for the period 8/15/2011 to 8/14/2014. I expect the outcomes to be several papers in refereed research journals.

As the budget for my NSF grant includes funds for supporting undergraduate and graduate research projects, I plan to attract 1–2 undergraduate students and one graduate research assistant to help with components of the proposed projects. These students and I will be part of a collaborative team together with Professor Connie Schober (the grant PI at the collaborating institution, the University of Central Florida) and her students.

During the course of the sabbatical I also plan to attend and speak at several international conferences. Furthermore, I propose to use the experience gained while at the National Science Foundation to explore curricular enhancements, and ways of attracting and retaining students to the Mathematical Sciences, and to help seek external funding to support such efforts.

Yours sincerely,

Annalisa Calini  
Professor  
Department of Mathematics

encl: Proposal, Letter of Invitation, Report for Previous Sabbatical, Curriculum Vitae
October 8, 2012

Professor Annalisa Calini
Department of Mathematics
College of Charleston
Charleston, SC 29424

Dear Annalisa,

It is a great pleasure to invite you to spend part of your sabbatical in the Fall of 2013 at the Department of Mathematics of the Pennsylvania State University. I greatly enjoyed your talk at the AMS meeting in University Park several years ago, and I have always been intrigued by your work on the vortex filament equation. So I look forward to discussing with you in depth on a variety of problems in integrable systems and its applications during your visit. Our department has a visitor program established by the late Dr. Shapiro which is designed to facilitate collaborative research between outstanding mathematicians from other institutions and Penn State faculty and students. I will nominate you for the Shapiro program, and I know that my colleagues Diane Henderson and Mark Levi are keen in having you here and will support this nomination.

Our department provides a congenial and stimulating research atmosphere and I anticipate that you will have a fruitful stay. I am happy to help with all necessary arrangements.

Sincerely,

Luen-Chau Li
Professor of Mathematics
luenli@math.psu.edu
I propose to work on the three projects described below. While my home base will be Charleston (allowing me to effectively direct students who will be involved in two of the projects), I plan an extended visit to Pennsylvania State University in Fall 2013, and several short visits to the University of Central Florida (UCF).

At Penn State, I will interact with faculty members in the Geometry and Integrable systems Group (including Professors Luu-Chau Li, Sergey Tabachnikov, and Mark Levi), and with Professor Diane Henderson, faculty leader of the W.G. Pritchard Fluid Mechanics Laboratory. I am applying for the Shapiro Visiting Program, which provides support for short-term visits of 4-8 weeks. Should my application be unsuccessful, I will use travel funds in my current NSF grant (see below) to fund a shorter visit of 2-3 weeks. While at Penn State, I will deliver a series of seminars on the Vortex Filament Equation (see Project 1), broaden my knowledge of and initiate new research collaborations in the area of Geometric Evolution Equations (see Project 2), and discuss lab-tank experiments to test and validate analytical and computational studies of rogue wave models conducted by Professor Connie Schober (UCF) and myself (see Project 3). A letter of invitation from Professor Li, indicating support for my nomination for the Shapiro Program, is included.

Visits to UCF are planned as part of an ongoing research project with Connie Schober, and will involve CofC students. This collaborative effort is funded by the National Science Foundation grant DMS-1109017, Collaborative RUI. Nonlinear Schrödinger Models in Fluid Dynamics: Rogue Waves and Vortex Filaments. 8/15/2011-8/14/2014. (See Projects 1. and 3., and section on Student Research.)

**Project 1: Vortex Filament Dynamics.** This project is centered on the Vortex Filament Equation (VFE), a model the self-induced dynamics of a tubular region of high vorticity within a perfect fluid largely at rest (e.g. a smoke ring moving through the air).

*Recent work and plans.* In our long-term collaboration, Tom Ivey (CofC) and I have exploited a well-known correspondence between the VFE and the Nonlinear Schrödinger (NLS) equation (see equation (1) below), a canonical model of nonlinear wave propagation in a variety of physical settings. We used this correspondence to study closed filaments associated with a family of special solutions of the NLS equation, known as *finite-gap* solutions and characterized by a "finite number of linear phases" in their expressions.

We have used a range of techniques including methods of algebraic geometry, perturbation analysis, and tools from topology and geometry, to relate geometric and topological properties—in particular, the knot types—of these solutions to the periodic spectra of the associated NLS solutions [3, 2].
In a recent collaboration with Stephane Lafortune and our former undergraduate student Scotty Keith, we proposed a general framework for studying the linear stability of VFE solutions (i.e., how robust these solutions are with respect to small perturbations of initial conditions). We applied this framework to filaments in the shape of small-amplitude torus knots [7], and filaments associated with NLS traveling wave solutions [5].

The proposed projects concern properties of periodic solutions of the VFE (closed vortex filaments), physically realistic extensions of the model, and new models of interacting vortex filaments:

- Determine the stability type of closed nearly-circular finite-gap filaments, by constructing a basis of solutions of the linearized VFE; and explore a generic mechanism of stability loss due to emergence of complex double points in the associated periodic spectra. (With T. Ivey and C. Schober.)

- Study extensions to physically more realistic models of vortex filament dynamics (e.g., incorporating thickness of the vortex core), and the emergence of geometric and topological complexity in the resulting near-integrable models. (With C. Schober and S. Lafortune.)

- Study a system of coupled NLS equations that model the dynamics of a pair of nearly-circular interacting vortex filaments. Tools will include numerical simulations and analytical techniques. (With C. Schober.)

**Project 2: Geometric Evolution Equations.** The VFE is a well-known integrable geometric evolution equation: geometric, as the velocity field is invariant under Euclidean motions (translation and rotation); integrable, since the induced equations for the curvature $\kappa$ and torsion $\tau$ of the moving curve possess an infinite number of conserved quantities, remaining constant throughout the evolution. As the vortex filament evolves, its total length is unchanged, and so are the integrals $\int \tau ds$ (total torsion), $\int \kappa^2 ds$ (total squared curvature), $\int \kappa^2 \tau ds$ (total helicity), and infinitely many others.

**Recent work and plans.** Gloria Marí-Beffa (U. of Wisconsin, Madison), Tom Ivey [11, 12], and I have begun to explore integrable curve evolutions that are invariant under more general groups, e.g., the group of affine transformations (the most general transformations of a plane or a space) into itself that send straight lines to straight lines. We discovered interesting connections with well-known integrable nonlinear models of water wave dynamics (including the Korteweg-de-Vries and Boussinesq equations).

During my visit at Penn State and the months following, I plan to broaden my knowledge of Geometric Evolution Equations, and hopefully begin new research collaborations. Two possible areas are:

- Discrete integrable systems arising in simple geometric contexts. One intriguing example is the *Pentagram Map*, recently discovered by R. Schwartz and a current research interest of Professor Tabachnikov at PSU [4]. This is a discrete integrable map on polygonal curves, which, in the continuous limit, corresponds to the Boussinesq equation, and should therefore be related to the curve evolution equations studied with Ivey and Marí-Beffa.

- Integrable discretizations of the VFE. This is a possible area of common interest with Professor Li, my host at Penn State, given his previous work in discrete integrable equations such as discrete NLS [8] and integrable spin systems [6], both naturally related to the VFE.
Project 3: Rogue Wave Models. Rogue waves are rare transient large amplitude waves whose heights are significantly larger than the background sea. In deep water, nonlinear wave focusing in localized regions of the sea is considered to be the main source of rogue wave development, and is related to the Benjamin-Feir (or modulational) instability, a process during which a uniform train of moderate amplitude waves loses energy to small perturbations of other waves of nearly the same frequency and direction. As a result, sidebands grow exponentially until nonlinear interactions become important, with nonlinear focusing producing further wave amplification.

Modulational instability is governed to leading order by the focusing nonlinear Schrödinger (NLS) equation

\[ i\psi_t + \psi_{xx} + 2|\psi|^2 = 0, \]

for the complex wave function \( \psi(x, t) \) (describing amplitude and phase of the wave as functions of space and time). A special class of NLS solutions, the homoclinic orbits of Stokes waves, contains large amplitude waves that are temporally and spatially localized.

\[ \text{Left: Surface plot of a 30m high wave reconstructed from satellite data. Right: One-mode homoclinic solution of the NLS equation.} \]

Recent work and plans. In our long-term collaboration, Connie Schober and I have addressed whether homoclinic solutions of the NLS equation are good models for physical rogue waves by examining their persistence under higher-order perturbations of the NLS equation (providing more accurate descriptions of the water wave dynamics), and by investigating whether rogue wave events from observational data can be correlated to proximity to homoclinic solutions [1, 9]. More recently [10], we proposed stability of homoclinic solutions as an additional selection criterion, and found that the stability requirement selects homoclinic orbits of maximal dimension.

During my sabbatical, and as part of our NSF-funded collaborative project, I propose to address the following questions:

- Persistence of large amplitude homoclinic structures in a higher order NLS model recently derived by fluid dynamics researchers Gramstad and Trulsen, with no artificial restrictions such as spatial evenness as in earlier studies. Persistent coherent structures would be more relevant as models of physically observable rogue waves.
Numerical simulations of a nonlinearly damped higher order NLS equation suggest that permanent downshifting (i.e., energy transfer from the dominant frequency to a lower dominant frequency) indicates sufficient cumulative damping to prevent further rogue wave generation. Is this a generic mechanism? Does nonlinear damping eliminate rogue waves through downshifting?

Can wave amplification due to mode coalescence be characterized in terms of the occurrence of higher order phase singularities? This can be useful in distinguishing categories of rogue waves, as has been done, e.g., for tropical storms.

Note: Connie has contacted Diane Henderson (PSU) about conducting lab-tank experiments to test and validate our analytical and numerical work. Connie plans to visit Dianne in Spring 2013 to initiate the experiments, and I plan to continue the interaction during my visit the following Fall.

**Student Research.** I plan to attract 1–2 undergraduate students and a graduate research assistant (funded through my NSF grant) to help with parts of Projects 1 and 3. Since my duties at the National Science Foundation end in August 2013, I plan to recruit the students during Fall 2013 and have them start on their research in early 2014. Connie and will form a collaborative team including all of our students. Two group meetings will take place, one at CoFC and one at UCF; internet communication and research tools will be used to help sustain the pace of the collaboration. This will provide a broader research training and help the students learn remote teamwork, an increasingly useful skill. We expect a particularly strong impact on the students from CoFC, as they will be able to learn about the research environment of a research university.

**Conferences.** I plan to attend and speak at several conferences including The Society for Industrial and Applied Mathematics Conference on *Nonlinear Waves and Coherent Structures*, and the *Nonlinear Evolution Equations and Dynamical Systems (NEEDS)* Conference, both held in the summer of even-numbered years.

**Curricular Initiatives.** During my time at the National Science Foundation, I have been involved in various initiatives supporting efforts to educate the next generation of mathematics and statistics students and improve the size and quality of the STEM workforce. During my sabbatical, I would like to explore related initiatives including curricular enhancements, and ways of attracting and retaining students to the Mathematical Sciences, and to help seek external funding to support such initiatives.

**Expected Outcomes.** By the end of the sabbatical I expect to have at least two papers submitted: one on vortex filament dynamics, and one on rogue waves; and several others in preparation. Some of these papers will be co-authored with students. I also hope to have new research directions in the area of geometric evolution equations.

**Salary-related Disclosure.** Note that my NSF appointment ends on August 27, 2013, and that I will still be receiving salary from the NSF during the first 12 days of the sabbatical leave.
REFERENCES


Updated report on my 2005-2006 sabbatical leave
Annalisa Calini, Department of Mathematics

This is a report on my sabbatical leave for the 2005-2006 academic year entitled Vortex Filament Models and Random Matrix Models, updated to describe subsequent publications and other consequences of the leave.

The leave was spent primarily at the University of Arizona, Tucson. During the period from summer 2005 to summer 2006, I also attended and delivered papers at three conferences and gave three seminar and colloquium presentations:


At the last of these, I also worked with Roberto Camassa of UNC Chapel Hill on the organization of the Special Session on Nonlinear Water Waves: Phenomena and Modeling. During my stay at the University of Arizona, I attended weekly meetings and a related reading course organized by the Random Matrix research group; I also regularly attended the Analysis and its Applications seminar, the departmental colloquium, and various other seminars.
Research projects

1. I continued to work on the Vortex Filament Equation in collaboration with Professor Tom Ivey (CoF). We gave a complete description of the knot types of closed vortex filaments "near" multiply-covered circles, using a perturbative step-by-step approach that constructs knotted filaments of increasing topological complexity.

2. In collaboration with Professor Joel Langer (CWRU), I completed a second article on Schwarzian reflection as a source of geometry on the infinite dimensional space $\Lambda$ of (un-parametrized) analytic curves in the complex plane. Schwarzian reflection generalizes complex conjugation (reflection in the real line) by taking a pair of elements in $\Lambda$ and reflecting the first curve into the second to produce a new element.

3. I worked with my doctoral advisor Nick Ercolani (UofA) on a rigorous proof of completeness of the NLS squared eigenfunctions for a finite-gap potential. Analogous completeness theorems are available for rapidly decaying potentials on the line; however, a full proof of completeness for the periodic finite-gap case is still missing and highly non-trivial, although such result is assumed and used throughout the literature.

Outcomes

1. During the first part of the sabbatical, I wrote and submitted a successful grant application to the National Science Foundation. This resulted in the:

   - National Science Foundation Grant DMS-0608587 ($150,000). (co-PI: T. Ivey.)
     
     RUI: Topology and Stability of Integrable Vortex Filament Motion.
     
     07/01/2006–06/30/2009.

The collaboration with Professor Tom Ivey resulted in the following article:


2. Professor Langer and I completed the following article:


3. Professor Ercolani and I wrote the following preprint.

   - A. Calini and N. Ercolani, Completeness of the squared eigenfunctions family for finite-gap solutions of the NLS equation.
In the course of the next several years, we realized that a general proof of completeness is not yet within reach, and we are now studying a restricted case. However, what we learnt from this project was instrumental in my recent work on stability with Professors Ivey, Lafortune, and Schober (see publications [16–18] in my CV).

During the year following the sabbatical, I gave two seminars and delivered four talks at international conferences based on research conducted during my sabbatical (see talks [36–41] in my CV).

Ideas developed during my sabbatical were explored in undergraduate- and master-level research:

Curriculum Vitae

Annalisa Maria Calini

Address
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College of Charleston,
66 George St., Charleston SC 29424-0001
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Phone: (843) 953-5732 Fax: (843) 953-1410

Date of Birth
15th of November, 1964

Citizenship
Dual: U.S. and Italian

Education

• University of Arizona, Tucson
  Ph.D. in Applied Mathematics (December 1994)
  Master of Sciences in Applied Mathematics (December 1992)

• Università degli Studi di Milano, Italy

Work Experience

• Program Director. Division of Mathematical Sciences, National Science Foundation, Arlington, VA (08/2011–Present).

• Professor. Department of Mathematics, College of Charleston, SC (08/2008–Present).

• Associate Professor. Department of Mathematics, College of Charleston, SC (08/2001–08/2008).


• Assistant Professor. Department of Mathematics, College of Charleston, Charleston, SC (08/1996–08/2001).

• Visiting Assistant Professor. Department of Mathematics, Case Western Reserve University, Cleveland, OH (01/1995–06/1996).

• Visiting Fellow. Special Program in Nonlinear Analysis, Centre for Mathematics and its Application. Australian National University, Canberra, Australia (10–12/1994).


Teaching Experience

• (August 1996–Present)

Courses taught: College Algebra, Contemporary Mathematics (for Liberal Arts majors), Elementary Statistics, Introduction to Abstract Mathematics, Discrete Structures, Advanced Calculus, Advanced Linear Algebra (Graduate), Continuous Mathematical Models, Vector and Tensor Analysis, Modern Differential Geometry (Graduate), Introduction to Partial Differential Equations (Undergraduate and Graduate), Capstone in Mathematics, Dynamical Systems (Graduate), Appreciation of Mathematics (Honors), Complex Variables (Graduate), Functional Analysis for Mathematical Physics (Senior level/Graduate).


• (August–December 1993) Teaching assistant for a geometry class for mathematics seniors and mathematics and mathematics education graduate students. Duties included teaching part of the lectures, grading, and supervising individual projects.


Students (College of Charleston)

• Elena Fenici (Math, MS 2011). On the Conserved Quantities of the Vortex Filament Equation. Master Thesis, co-directed with Dr. Stephane Lafortune. (NSF-funded.)

• J Seymour (Math undergraduate), Hunter Moss (Physics undergraduate). Stability of nonlinear waves. Undergraduate research project, co-directed with Dr. Stephane Lafortune. (NSF-funded.) (Summer 2011.)


• Kelly Epperson (Math, MS 2008). The Vortex Filament Equation and its Solutions and Energies. Master Thesis. (NSF-funded.)

• Kelly Epperson (Math, BS 2007). A Model of Vortex Filament Motion. Summer research project and Bachelor's Essay. (NSF-funded.) (05/2005–12/2006.)

• Cassel Sloan (Physics, BS 2005). Periodic Orbits in Triangular Billiards. Senior Thesis. (Spring 2005.)
• Evguenia (Jane) Ilina (Math, BS 2006). *Integrable Dynamics in Knotted Vortex Filaments*. Summer research project. (NSF-funded). (Summer 2003.)

• Kevin Young (Physics, BS 2005). *Integrable Dynamics in Knotted Vortex Filaments*. Summer research project. (NSF-funded). (Summer 2003.)

• Kelly Sweetingham (Math and Psychology, BS 2002). *Topics in Dynamical Systems*. Senior Thesis. (Spring 2002.)


**Publications in Refereed Journals**


Refereed Book Chapters


Conference Proceedings


Encyclopedia Articles


Submitted Articles


Work in Preparation

1. (with C.M. Schober) *Stability of rogue wave solutions in NLS models*.

2. (with S. Nelson) *Nearly circular interacting vortex filaments*.

3. (with J. Langer) *Schwarz Reflection Geometry III: the holonomy group and the pseudogroup of displacements*.

4. (with N. Ercolani) *Completeness of the squared eigenfunctions family for finite-gap solutions of the NLS equation*.
Other Works


Invited Presentations


52. *Four Lectures on an integrable equation describing the motion of vortex filaments: knotted solutions and their stability*. Escuela de Verano, Ecuaciones de Ondas Dispersivas, National University of Mexico (UNAM), Mexico City. June 4-8, 2012.


4. Mel’nikov Analysis for the NLS equation. The University of Notre Dame Symposium on Current and Future Directions in Applied Mathematics, University of Notre Dame, IN. April 18–21, 1996.


Grants and Awards

• Faculty Research and Development Grant-in-Time ($2,700). College of Charleston, Fall 2008.


• AWM-NFS Travel Grant ($1,200). The 4th International Conference on Geometry, Integrability and Quantization in Varna, Bulgaria, June 2002. (Declined as I received an NSF grant.)

• College of Charleston CETL Grant ($1,000). Course Development Grant: MATH 495, Capstone in Mathematics. Spring Semester 2002.

• College of Charleston In Praise of Teaching Award for promoting undergraduate research. 01/2002.


• Mathematics Department Research and Development Summer Grant ($1,000). University of Charleston, SC. June 1997.

• Research and Development Summer Grant ($2,250). University of Charleston, SC. 05/01–06/30, 1997.

• AWM-ONR award for participating in the AWM Workshop held at the SIAM Annual Meeting, Kansas City, MO. July 22–26, 1996.

• AWM-NFS travel grant ($1,000) for participating in the Summer School: The Painlevé Property: One Century Later, Cargése, France. June 3–22, 1996.

Research Interests

• Geometrical aspects of completely integrable PDEs.

• Chaos in finite and infinite dimensional Dynamical Systems.

• Relationship of geometry to the modelling of physical phenomena.

• Mathematical Physics.
Peer Referee for

Reviewer for

Conference Organization.


- Coorganizer with M. Q. Chen (The Citadel) and B. LeMesurier (CofC) of the *SIAM-SEAS Annual Meeting*, the Citadel and the College of Charleston, March 25–26, 2005.

- Coorganizer with Roberto Camassa (UNC) of the series (nicknamed *Cha-Cha Days*):
  