October 9, 2012

To: George Benson, President, College of Charleston
Re: Sabbatical proposal: *Vortex Filament Dynamics, Geometric Evolution Equations, and Rogue Wave Models.*

Dear President Benson,

I request a sabbatical leave for the full 2013-2014 academic year, to pursue work described in the attached proposal.

At the beginning of the leave, I plan to visit the Mathematics Department at Penn State University, to interact and collaborate with the Geometry and Integrable Systems Group and with Professor Diane Henderson, faculty leader of the W.G. Pritchard Fluid Mechanics Laboratory. A letter of invitation by Professor Luen-Chau Li is included.

I propose to work on three research projects, two of which are supported by grant DMS-1109017, *Collaborative RUI. Nonlinear Schrödinger Models in Fluid Dynamics: Rogue Waves and Vortex Filaments*, awarded by the National Science Foundation for the period 8/15/2011 to 8/14/2014. I expect the outcomes to be several papers in refereed research journals.

As the budget for my NSF grant includes funds for supporting undergraduate and graduate research projects, I plan to attract 1–2 undergraduate students and one graduate research assistant to help with components of the proposed projects. These students and I will be part of a collaborative team together with Professor Connie Schober (the grant PI at the collaborating institution, the University of Central Florida) and her students.

During the course of the sabbatical I also plan to attend and speak at several international conferences. Furthermore, I propose to use the experience gained while at the National Science Foundation to explore curricular enhancements, and ways of attracting and retaining students to the Mathematical Sciences, and to help seek external funding to support such efforts.

Yours sincerely,

Annalisa Calini
Professor
Department of Mathematics

encl: Proposal, Letter of Invitation, Report for Previous Sabbatical, Curriculum Vitae

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Department of Mathematics
Eberly College of Science

The Pennsylvania State University
University Park, PA 16802

October 8, 2012

Professor Annalisa Calini
Department of Mathematics
College of Charleston
Charleston, SC 29424

Dear Annalisa,

It is a great pleasure to invite you to spend part of your sabbatical in the Fall of 2013 at the Department of Mathematics of the Pennsylvania State University. I greatly enjoyed your talk at the AMS meeting in University Park several years ago, and I have always been intrigued by your work on the vortex filament equation. So I look forward to discussing with you in depth on a variety of problems in integrable systems and its applications during your visit. Our department has a visitor program established by the late Dr. Shapiro which is designed to facilitate collaborative research between outstanding mathematicians from other institutions and Penn State faculty and students. I will nominate you for the Shapiro program, and I know that my colleagues Diane Henderson and Mark Levi are keen in having you here and will support this nomination.

Our department provides a congenial and stimulating research atmosphere and I anticipate that you will have a fruitful stay. I am happy to help with all necessary arrangements.

Sincerely,

Luen-Chau Li

Luen-Chau Li
Professor of Mathematics
luenli@math.psu.edu

VORTEX FILAMENT DYNAMICS, GEOMETRIC EVOLUTION EQUATIONS, AND ROGUE WAVE MODELS

ANNALISA CALINI, DEPARTMENT OF MATHEMATICS

OCTOBER 9, 2012

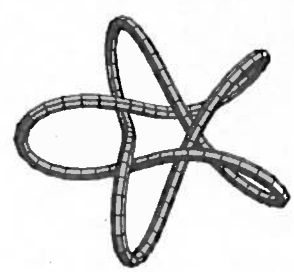
I propose to work on the three projects described below. While my home base will be Charleston (allowing me to effectively direct students who will be involved in two of the projects), I plan an extended visit to Pennsylvania State University in Fall 2013, and several short visits to the University of Central Florida (UCF).

At Penn State, I will interact with faculty members in the Geometry and Integrable systems Group (including Professors Luen-Chau Li, Sergey Tabachnikov, and Mark Levi), and with Professor Diane Henderson, faculty leader of the W.G. Pritchard Fluid Mechanics Laboratory. I am applying for the Shapiro Visiting Program, which provides support for short-term visits of 4-8 weeks. Should my application be unsuccessful, I will use travel funds in my current NSF grant (see below) to fund a shorter visit of 2-3 weeks. While at Penn State, I will deliver a series of seminars on the Vortex Filament Equation (see Project 1), broaden my knowledge of and initiate new research collaborations in the area of Geometric Evolution Equations (see Project 2), and discuss lab-tank experiments to test and validate analytical and computational studies of rogue wave models conducted by Professor Connie Schober (UCF) and myself (see Project 3). A letter of invitation from Professor Li, indicating support for my nomination for the Shapiro Program, is included.

Visits to UCF are planned as part of an ongoing research project with Connie Schober, and will involve CofC students. This collaborative effort is funded by the National Science Foundation grant DMS-1109017, *Collaborative RUI. Nonlinear Schrödinger Models in Fluid Dynamics: Rogue Waves and Vortex Filaments.* 8/15/2011–8/14/2014. (See Projects 1. and 3., and section on Student Research.)

Project 1: Vortex Filament Dynamics. This project is centered on the Vortex Filament Equation (VFE), a model the self-induced dynamics of a tubular region of high vorticity within a perfect fluid largely at rest (e.g. a smoke ring moving through the air).

Recent work and plans. In our long-term collaboration, Tom Ivey (CofC) and I have exploited a well-known correspondence between the VFE and the Nonlinear Schrödinger (NLS) equation (see equation (1) below), a canonical model of nonlinear wave propagation in a variety of physical settings. We used this correspondence to study closed filaments associated with a family of special solutions of the NLS equation, known as *finite-gap* solutions and characterized by a "finite number of linear phases" in their expressions.



A "finite-gap" knotted vortex filament.

We have used a range of techniques including methods of algebraic geometry, perturbation analysis, and tools from topology and geometry, to relate geometric and topological properties—in particular, the knot types—of these solutions to the periodic spectra of the associated NLS solutions [3, 2].

In a recent collaboration with Stephane Lafortune and our former undergraduate student Scotty Keith, we proposed a general framework for studying the linear stability of VFE solutions (i.e. how robust these solutions are with respect to small perturbations of initial conditions). We applied this framework to filaments in the shape of small-amplitude torus knots [7], and filaments associated with NLS traveling wave solutions [5].

The proposed projects concern properties of periodic solutions of the VFE (closed vortex filaments), physically realistic extensions of the model, and new models of interacting vortex filaments:

– Determine the stability type of closed nearly-circular finite-gap filaments, by constructing a basis of solutions of the linearized VFE; and explore a generic mechanism of stability loss due to emergence of complex double points in the associated periodic spectra. (With T. Ivey and C. Schober.)

– Study extensions to physically more realistic models of vortex filament dynamics (e.g. incorporating thickness of the vortex core), and the emergence of geometric and topological complexity in the resulting near-integrable models. (With C. Schober and S. Lafortune.)

– Study a system of coupled NLS equations that model the dynamics of a pair of nearly-circular interacting vortex filaments. Tools will include numerical simulations and analytical techniques. (With C. Schober.)

Project 2: Geometric Evolution Equations. The VFE is a well-known integrable geometric evolution equation: *geometric*, as the velocity field is invariant under Euclidean motions (translation and rotation); *integrable*, since the induced equations for the curvature κ and torsion τ of the moving curve possess an infinite number of *conserved quantities*, remaining constant throughout the evolution. As the vortex filament evolves, its total length is unchanged, and so are the integrals $\int \tau \mathrm{d}s$ (total torsion), $\int \kappa^2 \mathrm{d}s$ (total squared curvature), $\int \kappa^2 \tau \mathrm{d}s$ (total helicity), and infinitely many others.

Recent work and plans. Gloria Marí-Beffa (U. of Wisconsin, Madison), Tom Ivey [11, 12], and I have begun to explore integrable curve evolutions that are invariant under more general groups, e.g. the group of affine transformations (the most general transformations of a plane (or a space) into itself that send straight lines to straight lines). We discovered interesting connections with well-known integrable nonlinear models of water wave dynamics (including the Korteweg-de-Vries and Boussinesq equations).

During my visit at Penn State and the months following, I plan to broaden my knowledge of Geometric Evolution Equations, and hopefully begin new research collaborations. Two possible areas are:

– Discrete integrable systems arising in simple geometric contexts. One intriguing example is the *Pentagram Map*, recently discovered by R. Schwartz and a current research interest of Professor Tabachnikov at PSU [4]. This is a discrete integrable map on polygonal curves, which, in the continuous limit, corresponds to the Boussinesq equation, and should therefore be related to the curve evolution equations studied with Ivey and Marí-Beffa.

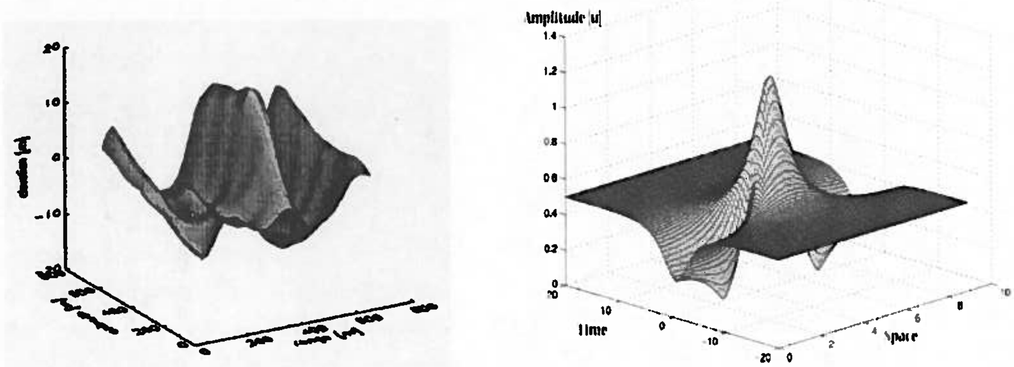
– Integrable discretizations of the VFE. This is a possible area of common interest with Professor Li, my host at Penn State, given his previous work in discrete integrable equations such as discrete NLS [8] and integrable spin systems [6], both naturally related to the VFE.

Project 3: Rogue Wave Models. Rogue waves are rare transient large amplitude waves whose heights are significantly larger than the background sea. In deep water, nonlinear wave focusing in localized regions of the sea is considered to be the main source of rogue wave development, and is related to the *Benjamin-Feir* (or *modulational*) *instability*, a process during which a uniform train of moderate amplitude waves loses energy to small perturbations of other waves of nearly the same frequency and direction. As a result, sidebands grow exponentially until nonlinear interactions become important, with nonlinear focusing producing further wave amplification.

Modulational instability is governed to leading order by the focusing nonlinear Schrödinger (NLS) equation

$$i\psi_t + \psi_{xx} + 2|\psi|^2 = 0, \tag{1}$$

for the complex wave function $\psi(x,t)$ (describing amplitude and phase of the wave as functions of space and time). A special class of NLS solutions, the *homoclinic orbits* of Stokes waves, contains large amplitude waves that are temporally and spatially localized.



Left: Surface plot of a 30m high wave reconstructed from satellite data. Right: One-mode homoclinic solution of the NLS equation.

Recent work and plans. In our long-term collaboration, Connie Schober and I have addressed whether homoclinic solutions of the NLS equation are good models for physical rogue waves by examining their *persistence* under higher-order perturbations of the NLS equation (providing more accurate descriptions of the water wave dynamics), and by investigating whether rogue wave events from observational data can be correlated to proximity to homoclinic solutions [1, 9]. More recently [10], we proposed *stability* of homoclinic solutions as an additional selection criterion, and found that the stability requirement selects homoclinic orbits of maximal dimension.

During my sabbatical, and as part of our NSF-funded collaborative project, I propose to address the following questions:

– Persistence of large amplitude homoclinic structures in a higher order NLS model recently derived by fluid dynamics researchers Gramstad and Trulsen, with no articial restrictions such as spatial evenness as in earlier studies. Persistent coherent structures would be more relevant as models of physically observable rogue waves.

– Numerical simulations of a nonlinearly damped higher order NLS equation suggest that permanent downshifting (i.e. energy transfer from the dominant frequency to a lower dominant frequency) indicates sufficient cumulative damping to prevent further rogue wave generation. Is this is a generic mechanism? Does nonlinear damping eliminate rogue waves through downshifting?

– Can wave amplification due to mode coalescence be characterized in terms of the occurrence of higher order phase singularities? This can be useful in distinguishing categories of rogue waves, as has been done, e.g., for tropical storms.

Note: Connie has contacted Diane Henderson (PSU) about conducting lab-tank experiments to test and validate our analytical and numerical work. Connie plans to visit Dianne in Spring 2013 to initiate the experiments, and I plan to continue the interaction during my visit the following Fall.

Student Research. I plan to attract 1–2 undergraduate students and a graduate research assistant (funded through my NSF grant) to help with parts of Projects 1 and 3. Since my duties at the National Science Foundation end in August 2013, I plan to recruit the students during Fall 2013 and have them start on their research in early 2014. Connie and will form a collaborative team including all of our students. Two group meetings will take place, one at CofC and one at UCF; internet communication and research tools will be used to help sustain the pace of the collaboration. This will provide a broader research training and help the students learn remote teamwork, an increasingly useful skill. We expect a particularly strong impact on the students from CofC, as they will be able to learn about the research environment of a research university.

Conferences. I plan to attend and speak at several conferences including The Society for Industrial and Applied Mathematics Conference on *Nonlinear Waves and Coherent Structures,* and the *Nonlinear Evolution Equations and Dynamical Systems (NEEDS)* Conference, both held in the summer of even-numbered years.

Curricular Initiatives. During my time at the National Science Foundation, I have been involved in various initiatives supporting efforts to educate the next generation of mathematics and statistics students and improve the size and quality of the STEM workforce. During my sabbatical, I would like to explore related initiatives including curricular enhancements, and ways of attracting and retaining students to the Mathematical Sciences, and to help seek external funding to support such initiatives.

Expected Outcomes. By the end of the sabbatical I expect to have at least two papers submitted: one on vortex filament dynamics, and one on rogue waves; and several others in preparation. Some of these papers will be co-authored with students. I also hope to have new research directions in the area of geometric evolution equations.

Salary-related Disclosure. Note that my NSF appointment ends on August 27, 2013, and that I will still be receiving salary from the NSF during the first 12 days of the sabbatical leave.

References

[1] A. Calini and C.M. Schober, *Homoclinic chaos increases the likelihood of rogue wave formation.* Physics Letters A, Vol. 298, No. 5–6 (2002), 335–349.

[2] A. Calini and T. Ivey, *Finite-gap Solutions of the Vortex Filament Equation: Genus One Solutions and Symmetric Solutions.* J. Nonlinear Sci., Vol. 15, no. 5 (2005), 321–361.

[3] A. Calini and T. Ivey, *Finite-gap Solutions of the Vortex Filament Equation II: Isoperiodic Deformations.* J. Nonlinear Sci., Vol. 17, no. 6 (2007), 527–567.

[4] V. Ovsienko, R. Schwartz and S. Tabachnikov, *The Pentagram map: a discrete integrable system* Comm. Math. Phys., 299 (2010), 409-446

[5] A. Calini, S.F. Keith and S. Lafortune, *Squared Eigenfunctions and Linear Stability Properties of Closed Vortex Filaments.* Nonlinearity **24**, no. 12 (2011), 33555-3583.

[6] L-C. Li and Z. Nie *Liouville Integrability of a Class of Integrable Spin Calogero-Moser Systems and Exponents of Simple Lie Algebras* Commun. in Math. Phys., **308**, no. 2 (2011), 415-438.

[7] A. Calini and T. Ivey, *Stability of Small-Amplitude Torus Knot Solutions of the Localized Induction Approximation.* J. Phys. A: Math. Theor. **44**, no. 33 (2011), 335204.

[8] L-C. Li and I. Nenciu, *The periodic defocusing AblowitzLadik equation and the geometry of Floquet CMV matrices* Advances in Math., **231** (2012), 3330-3388

[9] A. Calini and C.M. Schober, *Dynamical Criteria for Rogue Waves in NLS Models.* To appear in Nonlinearity (2012).

[10] A. Calini and C.M. Schober *Stability of rogue wave solutions in NLS models.* In preparation.

[11] A. Calini, T. Ivey, and G. Marí-Beffa, *Remarks on KdV-type Flows on Star-Shaped Curves.* Physica D Vol. 238, no. 8 (2009), 788–797.

[12] A. Calini, T. Ivey, and G. Marí-Beffa, *An integrable flow for starlike curves in centroaffine space.* Submitted to SIGMA, September 2012.

Updated report on my 2005-2006 sabbatical leave

Annalisa Calini, Department of Mathematics

This is a report on my sabbatical leave for the 2005-2006 academic year entitled **Vortex Filament Models and Random Matrix Models**, updated to describe subsequent publications and other consequences of the leave.

The leave was spent primarily at the University of Arizona, Tucson. During the period from summer 2005 to summer 2006, I also attended and delivered papers at three conferences and gave three seminar and colloquium presentations:

1. Conference presentation. *Finite-gap solutions of the Vortex Filament Equation.* FPU+50. Nonlinear waves 50 yeas after Fermi-Pasta-Ulam Conference. INSA de Rouen, France. June 21–25, 2005.

2. Colloquium. *Finite-gap solutions of the Vortex Filament Equation.* Applied Mathematics Colloquium. University of Arizona, Tucson, AZ. October 7, 2005.

3. Seminar. *Investigating knot types of algebro-geometric solutions of the Vortex Filament Equation.* Analysis and its Applications Seminar. University of Arizona, Tucson, AZ. October 11, 2005.

4. Seminar. *Finite-gap solutions of the Vortex Filament Equation.* Analysis Seminar. University of New Mexico, Albuquerque, NM. October 26, 2005.

5. Conference presentation. *Families of closed solutions of the integrable Vortex Filament Flow.* Conference on *Fluids and Waves–Recent Trends in Applied Analysis.* University of Memphis, Memphis, TN. May 11–13, 2006.

6. Conference presentation. *The formation of rogue waves in NLS models: persistence of homoclinic orbits.* The 6th AIMS International Conference on Dynamical Systems, Differential Equations and Applications. University of Poitiers, Poitiers, France. June 25–28, 2006.

At the last of these, I also worked with Roberto Camassa of UNC Chapel Hill on the organization of the Special Session on *Nonlinear Water Waves: Phenomena and Modeling.*

During my stay at the University of Arizona, I attended weekly meetings and a related reading course organized by the Random Matrix research group; I also regularly attended the Analysis and its Applications seminar, the departmental colloquium, and various other seminars.

Research projects

1. I continued to work on the Vortex Filament Equation in collaboration with Professor Tom Ivey (CofC). We gave a complete description of the knot types of closed vortex filaments "near" multiply-covered circles, using a perturbative step-by-step approach that constructs knotted filaments of increasing topological complexity.

2. In collaboration with Professor Joel Langer (CWRU), I completed a second article on *Schwarzian reflection* as a source of geometry on the infinite dimensional space Λ of (unparametrized) analytic curves in the complex plane. Schwarzian reflection generalizes complex conjugation (reflection in the real line) by taking a pair of elements in Λ and reflecting the first curve into the second to produce a new element.

3. I worked with my doctoral advisor Nick Ercolani (UofA) on a rigorous proof of completeness of the NLS *squared eigenfunctions* for a finite-gap potential. Analogous completeness theorems are available for rapidly decaying potentials on the line; however, a full proof of completeness for the periodic finite-gap case is still missing and highly non-trivial, although such result is assumed and used throughout the literature.

Outcomes

1. During the first part of the sabbatical, I wrote and submitted a successful grant application to the National Science Foundation. This resulted in the:

- National Science Foundation Grant DMS-0608587 ($150,000). (co-PI: T. Ivey.)
  RUI: Topology and Stability of Integrable Vortex Filament Motion.
  07/01/2006–06/30/2009.

The collaboration with Professor Tom Ivey resulted in the following article:

- A. Calini and T. Ivey, *Finite-gap Solutions of the Vortex Filament Equation II: Isoperiodic Deformations.* J. Nonlinear Sci., Vol. 17, no. 6 (2007), 527–567.

2. Professor Langer and I completed the following article:

- A. Calini and J. Langer, *Schwarzian Reflection Geometry II: Local and Global Behavior of the Exponential Map.* Experimental Mathematics, Vol. 16, no. 1 (2007), 321–346.

3. Professor Ercolani and I wrote the following preprint.

- A. Calini and N. Ercolani, *Completeness of the squared eigenfunctions family for finite-gap solutions of the NLS equation.*

In the course of the next several years, we realized that a general proof of completeness is not yet within reach, and we are now studying a restricted case. However, what we learnt from this project was instrumental in my recent work on stability with Professors Ivey, Lafortune, and Schober (see publications [16–18] in my CV).

During the year following the sabbatical, I gave two seminars and delivered four talks at international conferences based on research conducted during my sabbatical (see talks [36-41] in my CV).

Ideas developed during my sabbatical were explored in undergraduate- and master-level research:

- Kelly Epperson (Math BS 2007 and Math MS 2009): Bachelor's Essay and Master's Thesis.
- Scotty Keith (Math and Physics BS 2009): Senior Thesis.
- Sybil Prince Nelson (Math MS 2010): Master's Thesis.

Curriculum Vitae   Annalisa Maria Calini

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Date of Birth 15th of November, 1964

Citizenship Dual: U.S. and Italian

Education

- University of Arizona, Tucson
  Ph.D. in Applied Mathematics (December 1994)
  Dissertation: *Integrable Curve Dynamics.* Director: Nicholas Ercolani.
  Master of Sciences in Applied Mathematics (December 1992)

- Universitá degli Studi di Milano, Italy
  Laurea in Physics (equivalent of an MS). 110/110 Cum Laude (March 1989).
  Dissertation: *Influence of an External Signal on the Cooperative Dynamics of an Atomic Gas: Analytical Results.* Directors: Vincenzo G. Benza, Luigi Galgani.

Work Experience

- *Program Director.* Division of Mathematical Sciences, National Science Foundation, Arlington, VA (08/2011–Present).
- *Professor.* Department of Mathematics, College of Charleston, SC (08/2008–Present).
- *Associate Professor.* Department of Mathematics, College of Charleston, SC (08/2001–08/2008).
- *Director of Graduate Studies.* Department of Mathematics, College of Charleston, Charleston, SC (08/2002–07/2005 and 08–12/2006).
- *Assistant Professor.* Department of Mathematics, College of Charleston, Charleston, SC (08/1996–08/2001).
- *Visiting Assistant Professor.* Department of Mathematics, Case Western Reserve University, Cleveland, OH (01/1995–06/1996).
- *Visiting Fellow.* Special Program in Nonlinear Analysis, Centre for Mathematics and its Application. Australian National University, Canberra, Australia (10–12/1994).
- *Postdoctoral Fellow.* Special Semester in Dynamical Systems and Probabilistic Methods in Partial Differential Equations, Mathematical Sciences Research Institute. Berkeley, CA (01–07/1994).
- *Research Assistant.* University of Arizona (01/1991–12/1993).
- *Research Associate.* Los Alamos National Laboratory (07–08/1990). Project with J.C. Scovel and J.M. Hyman on symplectic numerical integrators.

- *Research Assistant.* University of Arizona (08/1989–06/1990). Collaboration with N. Ercolani, D.W. McLaughlin and C.M. Schober on spatial discretizations of the cubic focusing NLS equation, and extensions of the Melnikov method to the PDE setting.
- *Research Assistant.* University of Arizona (06–08/1989). Project with D.W. McLaughlin on analytical methods for detecting homoclinic chaos in a forced and damped NLS equation.

Teaching Experience

- (August 1996–Present)

  Courses taught: College Algebra, Contemporary Mathematics (for Liberal Arts majors), Elementary Statistics, Introduction to Abstract Mathematics, Discrete Structures, Advanced Calculus, Advanced Linear Algebra (Graduate), Continuous Mathematical Models, Vector and Tensor Analysis, Modern Differential Geometry (Graduate), Introduction to Partial Differential Equations (Undergraduate and Graduate), Capstone in Mathematics, Dynamical Systems (Graduate), Appreciation of Mathematics (Honors), Complex Variables (Graduate), Functional Analysis for Mathematical Physics (Senior level/Graduate).

  Independent Studies and Reading Courses: Classical Mechanics and Symplectic Geometry, Modern Differential Geometry for Physicists, Representation Theory of Lie Algebras.
- (January 1995–June 1996) Second semester Calculus for Science and Engineering majors, and Calculus for Liberal Arts majors, including Elementary Probability.
- (August–December 1993) Teaching assistant for a geometry class for mathematics seniors and mathematics and mathematics education graduate students. Duties included teaching part of the lectures, grading, and supervising individual projects.
- (August–December 1990) College Algebra.

Students (College of Charleston)

- Elena Fenici (Math, MS 2011). *On the Conserved Quantities of the Vortex Filament Equation.* Master Thesis, co-directed with Dr. Stephane Lafortune. (NSF-funded.)
- J Seymour (Math undergraduate), Hunter Moss (Physics undergraduate). *Stability of nonlinear waves.* Undergraduate research project, co-directed with Dr. Stephane Lafortune. (NSF-funded.) (Summer 2011.)
- Sybil Prince Nelson (Math, MS 2010). *Dynamics of Nearly Circular Vortex Filaments.* Master Thesis. (NSF-funded).
- Scott Forbes Keith (Math and Physics, BS 2009). *Linear Stability of Solutions of the Vortex Filament Equation.* Undergraduate research project and Senior Thesis. (NSF-funded). (05/2007–04/2009.)
- Kelly Epperson (Math, MS 2008). *The Vortex Filament Equation and its Solutions and Energies.* Master Thesis. (NSF-funded.)
- Kelly Epperson (Math, BS 2007). *A Model of Vortex Filament Motion.* Summer research project and Bachelor's Essay. (NSF-funded.) (05/2005–12/2006.)
- Cassel Sloan (Physics, BS 2005). *Periodic Orbits in Triangular Biliards.* Senior Thesis. (Spring 2005.)

- Evguenia (Jane) Ilina (Math, BS 2006). *Integrable Dynamics in Knotted Vortex Filaments.* Summer research project.(NSF-funded). (Summer 2003.)
- Kevin Young (Physics, BS 2005). *Integrable Dynamics in Knotted Vortex Filaments.* Summer research project. (NSF-funded). (Summer 2003.)
- Kelly Sweetingham (Math and Phsychology, BS 2002). *Topics in Dynamical Systems.* Senior Thesis. (Spring 2002.)
- Dimitre T. Milkov (Math and Economics, BS 2000). *Chaos in a Continuous-Time Model of Inventory Business Cycle.* Summer research project and Applied Mathematics Practicum. (06/1998–01/1999.)

Publications in Refereed Journals

18. (with C.M. Schober) *Dynamical Criteria for Rogue Waves in NLS Models.* To appear in Nonlinearity (2012).

17. (with S.F. Keith and S. Lafortune) *Squared Eigenfunctions and Linear Stability Properties of Closed Vortex Filaments.* Nonlinearity **24**, no. 12 (2011), 33555-3583.

16. (with T. Ivey) *Stability of Small-Amplitude Torus Knot Solutions of the Localized Induction Approximation.* J. Phys. A: Math. Theor. **44**, no. 33 (2011), 335204.

15. (with T. Ivey and G. Marí-Beffa) *Remarks on KdV-type Flows on Star-Shaped Curves.* Physica D Vol. 238, no. 8 (2009), 788–797.

14. (with J. Langer) *Schwarzian Reflection Geometry II: Local and Global Behavior of the Exponential Map.* Experimental Mathematics, Vol. 16, no. 1 (2007), 321–346.

13. (with T. Ivey) *Finite-gap Solutions of the Vortex Filament Equation II: Isoperiodic Deformations.* J. Nonlinear Sci., Vol. 17, no. 6 (2007), 527–567.

12. (with T. Ivey) *Finite-gap Solutions of the Vortex Filament Equation: Genus One Solutions and Symmetric Solutions.* J. Nonlinear Sci., Vol. 15, no. 5 (2005), 321–361.

11. (with J. Langer) *Schwarzian Reflection Geometry I: Continuous Iteration of Reflection.* Mathematische Zeitschrift Vol. 244, No. 4 (2003), 775–804.

10. (with C.M. Schober) *Homoclinic chaos increases the likelihood of rogue wave formation.* Physics Letters A, Vol. 298, No. 5–6 (2002), 335–349.

9. (with T. Ivey) *Connecting geometry, topology and spectra for finite-gap NLS potentials.* Physica D 152–153 (2001), 9–19.

8. (with T. Ivey) *Knot types, Floquet spectra, and finite-gap solutions of the Vortex Filament Equation.* Journal of Mathematics and Computers in Simulation. Vol. 55, No. 4–6 (2001), 341–350.

7. (with C.M. Schober) *Chaotic dynamics for a symmetry breaking perturbation of the NLS equation.* Journal of Mathematics and Computers in Simulation. Vol. 55, No. 4–6 (2001), 351–364.

6. *Recent developments in integrable curve dynamics.* In *Geometric Approaches to Differential Equations*, Ed. P. Vassiliou, I. Lisle. Australian Mathematical Society Lecture Series Vol. 15, Cambridge University Press (2000), 57–99.

5. (with T. Ivey) *Topology and sine-Gordon evolution of constant torsion curves.* Physics Letters A 254/3-4 (1999), 170–178.

4. (with C.M. Schober) *Mel'nikov analysis of a Hamiltonian perturbation of the Nonlinear Schrödinger Equation.* NATO Science Series C, Math. and Phys. Sci., Vol 533. Kluwer Academic Press (1999), 558–562.

3. (with T. Ivey) *Bäcklund transformations and knots of constant torsion.* Journal of Knot Theory and its Ramifications. Vol. 7, No. 6 (1998), 719–746.

2. (with N. Ercolani, D.W. McLaughlin, C.M. Schober) *Mel'nikov analysis of numerically induced chaos in the Nonlinear Schrödinger Equation.* Physica D 89 (1996), 227–260.

1. *A note on a Bäcklund transformation for the Continuous Heisenberg Model.* Physics Letters A 203 (1995), 333–344.

Refereed Book Chapters

1. (with C.M. Schober) *Rogue Waves in High-Order Nonlinear Schrdinger Models.* In *Extreme Ocean Waves*, Pelinovsky, E.; Kharif, C. Editors, Springer-Verlag (2008), 31–52.

Conference Proceedings

1. *Integrable Dynamics of Knotted Vortex Filaments.* Four invited lectures in Proceedings of the Fifth International Conference on Geometry, Integrability and Quantization, I.M. Mladenov and G.L. Naber Editors (2004), 11–50.

Encyclopedia Articles

1. *The Mel'nikov Method.* In *Encyclopedia of Nonlinear Science.* Ed. Alwyn Scott. New York: Routledge, 2005.

2. *Elliptic Functions.* In *Encyclopedia of Nonlinear Science.* Ed. Alwyn Scott. New York: Routledge, 2005.

Submitted Articles

1. (with T. Ivey and G. Marí-Beffa) *An integrable flow for starlike curves in centroaffine space.* Submitted to SIGMA, September 2012.

Work in Preparation

1. (with C.M. Schober) *Stability of rogue wave solutions in NLS models.*

2. (with S. Nelson) *Nearly circular interacting vortex filaments.*

3. (with J. Langer) *Schwarz Reflection Geometry III: the holonomy group and the pseudogroup of displacements.*

4. (with N. Ercolani) *Completeness of the squared eigenfunctions family for finite-gap solutions of the NLS equation.*

Other Works

- (with G. Kovacic and C.M. Schober) *Multiphase Solutions of the Maxwell-Bloch Equation.* Unpublished (2000).
- (with C.M. Schober) *Chaotic Dynamics for Symmetry-Breaking Perturbations of Integrable Equations.* Contributed Abstract for the XXIII European Geophysical Society General Assembly, Nice, France, 20-24 April 1998. Annales Geophysical Supplement, Vol. 16, 1998.
- (with P. D. Miller) *Modulational stability in the focusing Ablowitz-Ladik equations.* Unpublished (1996).
- *Integrable Curve Dynamics.* Ph.D. Thesis. University of Arizona, December 1994.
- *Influenza di un Segnale Esterno sulla Dinamica Cooperativa di un Gas Atomico: Risultati Analitici.* Tesi di Laurea. Dipartimento di Fisica, Universitá degli Studi di Milano, March 1989.

Invited Presentations

54. *Knotted solutions of the vortex filament equation and their stability.* Nonlinear Evolution Equations and Dynamical Systems, NEEDS 2012, Conference. Orthodox Academy of Crete, Kolymbari, Greece. July 8-15, 2012.

53. *On the stability of closed vortex filaments.* Special Session on *Applied Analysis and Dynamical in Engineering and Sciences*; the 9th AIMS Conference on Dynamical Systems, Differential Equations, and Applications. Orlando, FL. July 1-5, 2012.

52. *Four Lectures on an integrable equation describing the motion of vortex filaments: knotted solutions and their stability.* Escuela de Verano, Ecuaciones de Ondas Dispersivas, National University of Mexico (UNAM), Mexico City. June 4-8, 2012.

51. *Integrable evolution of closed vortex filaments: finite-gap solutions and their linear stability.* ESF-ERCOM International Conference: *Knots and Links: From Form to Function.* Mathematical Research Center (CRM) Ennio De Giorgi Pisa, Italy. July 2-8, 2011.

50. *Linear stability of closed finite-gap solutions of the Vortex Filament Equation.* Special Session on *Geometry of Drops, Membranes and Filaments.* The 7th IMACS International Conference on Nonlinear evolution equations and wave phenomena: computation and theory. Athens, GA. April 4–7, 2011.

49. *Integrable evolution of closed vortex filaments: finite-gap solutions and their linear stability.* (Plenary Speaker.) The 45th Texas Geometry and Topology Conference, Texas Tech University, February 18-20, 2011.

48. *A Study of Stability of Rogue Wave Solutions of the Nonlinear Schrödinger Equation.* Special Session on *Applied Analysis and Dynamics in Engineering and Sciences.* The 8th AIMS Conference on Dynamical Systems, Differential Equations and Applications. Dresden University of Technology. Dresden, Germany. May 25–28, 2010.

47. *Linear stability of small-amplitude torus knot solutions of the Vortex Filament Equation.* Applied Mathematics Seminar. University of South Carolina, SC. April 21, 2010.

46. *Linear stability of small-amplitude torus knot solutions of the Vortex Filament Equation.* Special Session on *Geometric Flows, Moving Frames and Integrable Systems.* AMS Central Spring Section Meeting, St. Paul, MN. April 10–11, 2010.

45. *From circles to cables: closed finite-gap solutions of the Vortex Filament Flow.* Special Session on *Integrable Systems and Related Areas.* AMS Fall Eastern Sectional Meeting, State College, PA, October 24-25, 2009.

44. *Remarks on KdV-type Flows on Star-Shaped Curves. Solitons in the Roaring Forties*, CPNLW09 (Coherence and Persistence in Nonlinear Waves) Conference, Nice University, Campus Valrose. France. January 6–9, 2009.

43. *The Formation of Rogue Waves in Nonlinear Schroedinger Models.* Physics Department Colloquium. College of Charleston. October 30, 2008.

42. *Finite-Gap Solutions of the Vortex Filament Flow: Isoperiodic Deformations.* Special Session on *Periodic Problems in Soliton Equations.* International Conference on Nonlinear Waves–Theory and Applications. Beijing, China. June 9–12, 2008.

41. *Cable Formation for Finite-Gap Solutions of the Vortex Filament Flow.* Special Session on *New trends in Spectral Analysis and PDEs.* First Joint Meeting of the AMS-NZMS. Victoria University of Wellington, New Zealand. December 12–15, 2007.

40. *Investigating knot types of finite-gap solutions of the Vortex Filament Equation.* NEEDS 2007 Workshop on Nonlinear Evolution Equations and Dynamical Systems. L'Ametlla de Mar (Spain). June 15–24, 2007.

39. *The Localized Induction Equation for Vortex Filament Motion: Families of Closed Solutions and their Properties.* Special Session on *Solitons and Nonlinear Patterns on Closed Surfaces/Curves.* The 5th IMACS International Conference on Nonlinear evolution equations and wave phenomena: computation and theory. Athens, GA. April 16–19, 2007.

38. *Investigating knot types of finite-gap solutions of the Vortex Filament Equation.* Differential Equations Seminar. North Carolina State University. March 23, 2007.

37. *Closed solutions of the vortex filament flow I.* Applied Mathematics Seminar. University of Wisconsin, Madison. March 7, 2007.

36. *Finite-gap Solutions of the Localized Induction Equation for Vortex Filaments: Their Geometry and Topology.* (Speaker and coorganizer.) Minisymposium on *Generation and Dynamics of Vortex Filaments: Geometrical, Topological, Analytical, and Experimental Approaches.* SIAM Conference on Nonlinear Waves and Coherent Structures. University of Washington, Seattle, WA. September 9–12, 2006.

35. *The formation of rogue waves in NLS models: persistence of homoclinic orbits.* (Speaker and coorganizer.) Special session on *Nonlinear water waves: phenomena and modeling.* The 6th AIMS International Conference on Dynamical Systems, Differential Equations and Applications. University of Poitiers, Poitiers, France. June 25–28, 2006.

34. *Families of closed solutions of the integrable Vortex Filament Flow.* Conference on *Fluids and Waves–Recent Trends in Applied Analysis.* University of Memphis, Memphis, TN. May 11–13, 2006.

33. *Finite-gap solutions of the Vortex Filament Equation.* Analysis Seminar. University of New Mexico, Albuquerque, NM. October 26, 2005.

32. *Investigating knot types of algebro-geometric solutions of the Vortex Filament Equation.* Analysis and its Applications Seminar. University of Arizona, Tucson, AZ. October 11, 2005.

31. *Finite-gap solutions of the Vortex Filament Equation.* Applied Mathematics Colloquium. University of Arizona, Tucson, AZ. October 7, 2005.

30. *Finite-gap solutions of the Vortex Filament Equation.* (Contributed talk.) FPU+50. Nonlinear waves 50 yeas after Fermi-Pasta-Ulam Conference. INSA de Rouen, France. June 21–25, 2005.

29. *Knotted vortex filaments: finding connections between topology and spectra.* (Invited poster presentation.) DMac Fest. Conference in honor of Dave McLaughlin's 60th birthday. Chapel Hill, NC. October 23–24, 2004.

28. *Knotted vortex filaments: finding connections between topology and spectra.* (Contributed talk.) SIAM Conference on Nonlinear Waves and Coherent Structures, Orlando, FL. October 2–5, 2004.

27. *Chaotic direction reversing waves in a perturbed nonlinear Schrödinger equation.* MIDIT Seminar. Danish Technical University. Lyngby, Denmark. May 25, 2004.

26. *Knotted vortex filaments: finding connections between topology and spectra.* Applied Mathematics Colloquium. University of North Carolina, Chapel Hill, NC. April 16, 2004.

25. *Knotted vortex filaments: finding connections between topology and spectra.* MIDIT Seminar. Institute for Mathematical Modelling (IMM), Danish Technical University. Lyngby, Denmark. June 24, 2003.

24. *Four Lectures on Curve Geometry and Soliton Theory.* (Principal speaker.) The *Fifth International Conference on Geometry, Integrability and Quantization.* Varna, Bulgaria, June 6–15, 2003.

23. *Integrable Dynamics of Knotted Vortex Filaments.* PDE Seminar. University of Missouri. Columbia, MO. October 10, 2002.

22. *A Continuous Analogue of Iteration of Schwarz Reflection.* Mathematics Department Colloquium. University of Missouri. Columbia, MO. October 10, 2002.

21. *Homoclinic Chaos Increases the Likelihood of Rogue Wave Formation.* MIDIT Seminar. Institute for Mathematical Modelling (IMM), Danish Technical University. Lyngby, Denmark. May 30, 2002.

20. *Curve Geometry and Soliton Theory.* MIDIT Seminar. Institute for Mathematical Modelling (IMM), Danish Technical University. Lyngby, Denmark. July 3, 2001.

19. *Mel'nikov analysis of a symmetry-breaking perturbation of the NLS equation.* (Invited poster presentation.) Nonlinear Science Festival III. Technical University of Denmark. Lyngby, Denmark, June 12–15, 2001.

18. *Curve Geometry and Soliton Theory.* Mathematics Department Colloquium. Old Dominion University. Norfolk, VA. April 13, 2001.

17. *Curve Geometry and Soliton Theory.* Mathematics Research Seminar. Saint Mary's College of Maryland. Saint Mary's City, MD. April 12, 2001.

16. *Connecting geometry, topology and spectra for finite-gap NLS potentials.* (Invited poster presentation.) Nonlinear Analysis 2000. Courant Institute of Mathematical Sciences. New York, NY. May 28–June 2, 2000.

15. *Curve Geometry and Soliton Theory.* Integrating Integrability into Mathematics and Science. Conference in honor of Vladimir Zacharov's 60th birthday. Tucson, AZ. October 29–31, 1999.

14. *Topological changes near homoclinic solutions of an integrable model of vortex filament evolution.* (Speaker and coorganizer.) Minisymposium on *Near-singular Phenomena in Conservative Wave Equations and their Perturbations.* ICIAM99, International Congress on Industrial and Applied Mathematics, Edinburgh, Scotland. July 5–9, 1999.

13. *Topology and Sine-Gordon Evolution of Constant Torsion Curves.* Special Session on *Theoretical Aspects of Solitons and Integrability.* The 1999 IMACS International Conference on Nonlinear Evolution Equations and Wave Phenomena: Computation and Theory. Athens, GA. April 12–19, 1999.

12. *Integrable Systems Methods in the Geometry and Topology of Evolving Curves.* Southeastern Geometry Conference, University of Georgia, Athens, GA. April 10–11, 1999.

11. *Topology and Sine-Gordon Evolution of Constant Torsion Curves.* Special Session on *Filaments, Interfaces and Patterns.* 1998 Fall Western Section Meeting, Tucson, AZ, November 13–15, 1998.

10. *Integrable Systems Methods in Curve Evolution.* Mathematical Sciences Department Seminar, Montana State University. Bozeman, MT. April 3, 1998.

9. *Mel'nikov Analysis for the NLS equation.* Mathematics Department Colloquium. Ball State University, IN. March 5, 1998.

8. *Bäcklund Transformations of Knots of Constant Torsion.* Mathematics Department Colloquium. Rensselaer Polytechnic Institute, Troy, NY. July 14, 1997.

7. *Bäcklund Transformations of Knots of Constant Torsion.* Applied Mathematics and PDE Seminar. University of Notre Dame, IN. June 3, 1997.

6. *Bäcklund Transformations of Knots of Constant Torsion.* (Speaker and coorganizer.) Minisymposium on *Integrable Systems Methods for Curve Evolution.* SIAM Conference on Applications of Dynamical Systems, Snowbird, UT. May 18–22, 1997.

5. *Soliton Equations and Evolution of Space Curves.* (Speaker and coorganizer.) Minisymposium on *Nonlinear Dynamics and Geometry in Integrable and Near-Integrable partial Differential Equations.* SIAM Annual Meeting, Kansas City, MO. July 22–26, 1996.

4. *Mel'nikov Analysis for the NLS equation.* The University of Notre Dame Symposium on *Current and Future Directions in Applied Mathematics*, University of Notre Dame, IN. April 18–21, 1996.

3. *Dynamics of Curves and Integrable Equations.* (Principal Speaker.) Miniworkshop on *Geometry and Differential Equations.* University of Canberra, Australia. May 15–16, 1995.

2. *Bäcklund Transformations and Immersed Knots. Integrable evolution of spherical curves and the Nonlinear Schrödinger Equation: a geometrical point of view.* Geometry and Topology Seminar. University of Maryland at College Park, MD. March 8, 1995.

1. *Dynamics of Curves.* Arizona Days, Los Alamos National Laboratories, Los Alamos, NM. February 5–6, 1994.

Grants and Awards

- National Science Foundation Grant DMS-1109017 ($174,104) (PI). *Collaborative RUI. Nonlinear Schrödinger Models in Fluid Dynamics: Rogue Waves and Vortex Filaments.* Collaborating PI: C.M. Schober, UCF. 8/15/2011–8/14/2014.

- Faculty Research and Development Grant-in-Time ($2,700). College of Charleston, Fall 2008.
- National Science Foundation Supplemental Grant DMS-0853273 ($8,000). (PI, co-PI: T. Ivey.) *Supplement. RUI: Topology and Stability of Integrable Vortex Filament Motion.* 09/01/2008–06/30/2009.
- National Science Foundation Grant DMS-0739409 ($36,324). (PI, co-PI: C.M. Schober, UCF.) *Collaborative Proposal: Southeastern Atlantic Mathematical Sciences Workshop.* 09/01/2008–08/31/2011.
- National Science Foundation Grant DMS-0739386 ($12,108). (PI, co-PI: C.M Schober, UCF.) *Collaborative Proposal: Southeastern Atlantic Mathematical Sciences Workshop: 2007 Meeting.* 09/01/2007–08/31/2008.
- National Science Foundation Grant DMS-0608587 ($150,000). (PI, co-PI: T. Ivey.) *RUI: Topology and Stability of Integrable Vortex Filament Motion.* 07/01/2006–06/30/2009.
- National Science Foundation Grant DMS-0407843 ($29,992). (PI, co-PIs: L-S. Luo, NIA and ODU, and C.M. Schober, UCF.) *Collaborative Proposal: Southeastern Applied Mathematics Days.* 09/01/2004–08/31/2007.
- National Science Foundation grant DMS-0204557 ($148,000). (PI, co-PI: T. Ivey.) *RUI: Integrable Dynamics of Knotted Vortex Filaments.* 07/15/2002–06/30/2005.
- AWM-NFS Travel Grant ($1,200). *The 4th International Conference on Geometry, Integrability and Quantization* in Varna, Bulgaria, June 2002. (Declined as I received an NSF grant.)
- College of Charleston CETL Grant ($1,000). Course Development Grant: MATH 495, Capstone in Mathematics. Spring Semester 2002.
- College of Charleston *In Praise of Teaching Award* for promoting undergraduate research. 01/2002.
- National Science Foundation grant DMS-9705005 ($56,531) (PI). *Integrable Dynamics of Knotted Vortex Filaments.* 07/15/1997–06/30/2001.
- Mathematics Department Research and Development Summer Grant ($1,000). University of Charleston, SC. June 1997.
- Research and Development Summer Grant ($2,250). University of Charleston, SC. 05/01–06/30, 1997.
- AWM-ONR award for participating in the AWM Workshop held at the SIAM Annual Meeting, Kansas City, MO. July 22–26, 1996.
- AWM-NFS travel grant ($1,000) for participating in the Summer School: *The Painlevé Property: One Century Later*, Cargése, France. June 3–22, 1996.

Research Interests

- Geometrical aspects of completely integrable PDEs.
- Chaos in finite and infinite dimensional Dynamical Systems.
- Relationship of geometry to the modelling of physical phenomena.
- Mathematical Physics.

Peer Referee for

Physica Scripta, Journal of Physics A, Nonlinearity, Physica D, SIAM Journal of Applied Mathematics, Pacific Journal of Mathematics, Journal of Nonlinear Science, Journal of Discrete and Continuous Dynamical Systems, Journal of Mathematics and Computers in Simulation, Journal of Differential Equations, Physical Review E.

Reviewer for

MathReviews (1996–2002, 2007), National Science Foundation (Panelist, DMS-Applied Mathematics, 2006, 2007, 2008, 2009, 2010). European Science Foundation (Grant Reviewer, 2009, 2010).

Conference Organization.

- Coorganizer with Alex Kasman (CofC) of the *Eleventh Southeastern Geometry Conference*, College of Charleston, March 24–6, 2000.
- Coorganizer with Alex Kasman and Thomas Ivey (both at CofC) of the *Fourtheen Southeastern Geometry Conference*, College of Charleston, March 28–30, 2003.
- Advisory board member (2003–2007) for the *International Conference on Geometry, Integrability and Quantization*. Varna, Bulgaria.
- Coorganizer with M. Q. Chen (The Citadel) and B. LeMesurier (CofC) of the *SIAM-SEAS Annual Meeting*, the Citadel and the College of Charleston, March 25–26, 2005.
- Coorganizer with Roberto Camassa (UNC) of the series (nicknamed *Cha-Cha Days*):

  SEAMS Worshop 2004, College of Charleston, September 17–19, 2004.
  SEAMS Worshop 2005, University of North Carolina, Chapel Hill, September 23–25, 2005.
  SEAMS Worshop 2006, College of Charleston, September 29–October 1, 2006.
  SEAMS Workshop 2007, National Institute of Aerospace, October 19–21, 2007.
  SEAMS Workshop 2008, University of North Carolina, Chapel Hill, October 31–November 2, 2008.
  SEAMS Workshop 2009, University of Central Florida, November 6–8, 2009.
  SEAMS Workshop 2010, College of Charleston, September 24–26, 2010.